NOTATION

d = diameter of the nozzle, cm
D = diffusion coefficient of ethyl salicylate in water, cm^2/s
k = local mass transfer coefficient at radial location x, cm/s
t = elapsed time from commencement of transfer, s
t' = constant rate period predicted by the methods of Macleod and Todd, s
u = nozzle exit velocity, cm/s
x = radial distance from stagnation point, cm
Re = Reynolds number, ud/ν
Sc = Schmidt number for ethyl salicylate/water system ν/D
Sh = Sherwood number, kd/D

Greek Letters

ν = kinematic viscosity of water, cm^2/s

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An Analysis of Turbulent Pipe Flow with Viscosity Variation in the Wall Region

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A paper and notes by the author (1971, 1972, 1973) showed that wall region and transition region mass transfer for turbulent pipe flow are represented by the equations

$$k^+_{EW} = 0.065\ N_{Sc}^{-2/3} \tag{1}$$

and

$$k^+_{ET} = 0.0615\ N_{Sc}^{-1/2} \tag{2}$$

These equations are shown to be consistent with the wall region frequency relationship

$$\frac{u^{*2}\,t}{\nu} = 338 \tag{3}$$

This note presents an extension of this work to heat transfer with variable viscosity in the wall region.

NEWTONIAN FLUIDS

Equation (2) corresponds to the penetration model with the frequency from Equation (3). This can be expressed in heat transfer terms to represent the transition region properties

$$\frac{h}{\rho_T\, C_{pT}} = 2\sqrt{\frac{k_T}{\pi \rho_T\, C_{pT}\, t}} \tag{4}$$

Substitution of Equation (3) which represents bulk fluid properties provides

$$\frac{h}{\rho_T\, C_{pT}} = 0.0615\ u^*\,(N_{PrT})^{-1/2}\,(\nu_T/\nu_b)^{1/2} \tag{5}$$

Equation (2) reduces to the mass transfer analog of Equation (5) for isothermal conditions. Equation (5) would be expected to represent the transition region and to be applicable to heat transfer data with significant resistance in this region.

The momentum analog is useful in evaluating the kinematic viscosity ratio for Equation (5). Isothermal turbulent flow results in an essentially constant kinematic viscosity in the wall and transition regions. Momentum equations for isothermal and nonisothermal flow are

$$\frac{\tau}{\rho} = 0.332\ U^2_{\infty i}\sqrt{\frac{\nu}{U_{\infty i}\, x}} \tag{6}$$

and

$$\frac{\tau}{\rho} = 0.332\ U^2_{\infty ni}\sqrt{\frac{\nu_T}{U_{\infty ni}\, x}} \tag{7}$$

Combination of Equations (3), (6), and (7) yields

$$U^+_{\infty ni} = (\nu_b/\nu_T)^{1/3}\ U^+_{\infty i} \tag{8}$$

with the assumption that Equation (3) represents the bulk kinematic viscosity. The dimensionless shearing stress for a developing boundary layer on a flat plate leads to the friction factor relationship

$$f = \frac{0.664}{\sqrt{\dfrac{U_\infty x}{\nu}}} \tag{9}$$

Friction factor equations for isothermal and nonisothermal flow with substitution of Equation (3) are

$$f_i = \frac{0.036}{\sqrt{U^+_{\infty ni}}} \tag{10}$$

$$f_{ni} = \frac{0.036}{\sqrt{U^+_{\infty ni}\ \nu_b/\nu_T}} \tag{11}$$

Combination of Equations (10) and (11) with Equation (8) results in

$$f_i = (\nu_b/\nu_T)^{2/3} f_{ni} \tag{12}$$

Kreith and Summerfield (1949, 1950) report experimental data for heating of water and n-butanol which show friction factors that are less than for isothermal flow at the bulk fluid conditions. These friction factor data can be empirically correlated with the ratio of the wall surface and bulk kinematic viscosities as shown by Figure 1. The correlating equation is

$$\frac{f_i}{f_{ni}} = 0.1\,(\nu_b/\nu_{ws} - 1)^{1.1} + 1 \tag{13}$$

Equations (12) and (13) then combine to provide an estimate of the effective wall region kinematic viscosity.

$$\nu_w = \frac{\nu_b}{[0.1\,(\nu_b/\nu_{ws} - 1)^{1.1} + 1]^{3/2}} \tag{14}$$

This kinematic viscosity with density, specific heat, and the Prandtl number for the temperature corresponding to this viscosity should then apply to the heat transfer Equation (5).

Kreith and Summerfield (1950) also report experimental heat transfer data for heating of n-butanol. These data represent resistance of the laminar boundary and transition regions; therefore, it is necessary to separate the transition region heat transfer for comparison with Equation (5). Eddy diffusivity for the laminar boundary region is

$$\frac{\epsilon}{\nu} = 0.0049\, y^{+3}$$

in accordance with Equation (1). Integration of the sum of the molecular and eddy thermal conductivities for the region $0 < y^+ < 2$ and application to the corresponding boundary layer thickness provides the heat transfer equation for this boundary layer.

$$\frac{h_{BL}}{\rho\, C_p} = (1/N_{Pr} + 0.0098)u^* \tag{15}$$

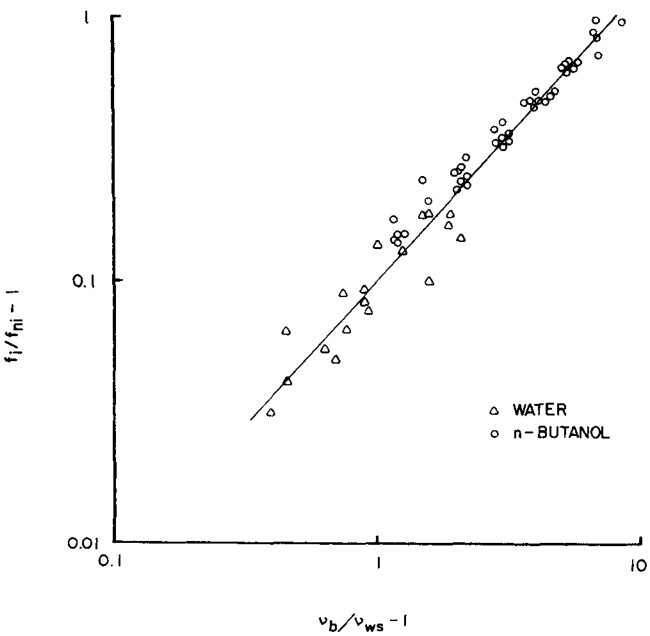

Fig. 1. Nonisothermal friction factor correlation.

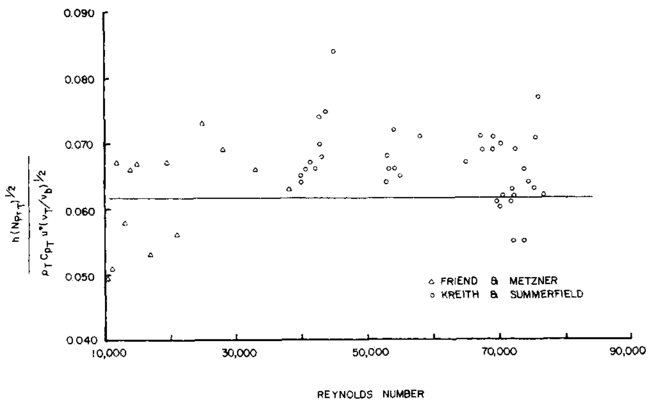

Fig. 2. Nonisothermal heat transfer data.

The total heat transfer resistance is then the sum of the boundary layer resistance from Equation (15) and the transition region resistance from Equation (5). Equation (15) is assumed to correspond to physical properties at the wall surface temperature. Figure 2 shows a comparison of the n-butanol data with Equation (5). Heat transfer data for molasses and corn syrup with the Prandtl number range of 55 to 190 are reported by W. L. Friend and Metzner (1958). These data are also shown by Figure 2. Reasonable agreement is observed between Equation (5) and the experimental data.

NON-NEWTONIAN FLUIDS

Power law non-Newtonian fluids show an apparent viscosity variation in the wall region which can be treated similarly to viscosity variation with temperature for Newtonian fluids. The apparent viscosity for a power law fluid is expressed as

$$\mu_a = K\, u^* \left(\frac{du}{dy}\right)^{n-1} \tag{16}$$

The ratio of the apparent viscosity near the wall to the apparent viscosity at the wall can be obtained from Equation (16)

$$\frac{\mu_a}{\mu_{aw}} = \left[\frac{(du/dy)}{(du/dy)_w}\right]^{n-1} = \left[\frac{du^+/dy^+}{(du^+/dy^+)_w}\right]^{n-1}$$

For the region $0 < y^+ < 2$, $du^+/dy^+ = 1$ and with the assumption of constant fluid density,

$$\nu_a = \nu_{aw}\,(du^+/dy^+)^{n-1} \tag{17}$$

The friction factor relationship from Equation (12) is then

$$f_N = [(du^+/dy^+)^{n-1}]^{2/3} f_P \tag{18}$$

Popovich and Hummel (1967) report that $y^+ = 12.1$ represents the transition between predominantly laminar and turbulent conditions. The velocity profile slope, $du^+/dy^+ = 0.41$, at $y^+ = 12.1$ could be expected to apply for Equation (18). The Reynolds number could also be expected to apply to this turbulent boundary condition

$$N_{Re} = \frac{DV}{\nu_{aw}\,(du^+/dy^+)^{n-1}} = \frac{DV}{\nu_{aw}(0.41)^{n-1}} \tag{19}$$

Equation (19) reduces to the conventional Reynolds number for isothermal Newtonian fluid flow with $n = 1$ and $\nu_b = \nu_{aw}$. The conventional Reynolds number with the bulk temperature kinematic viscosity is also applicable to nonisothermal Newtonian fluid flow because the temperature gradient occurs between the tube wall and $y^+ = 12.1$

at low and moderate heat fluxes. Thus Equation (19) could also be expected to apply for nonisothermal power law non-Newtonian fluid flow with use of the bulk temperature properties.

Friend and Metzner (1959) report friction factor and heat transfer data for several power law non-Newtonian fluids. Low temperature differences between the pipe wall and fluid were used so that relatively little viscosity variation will result from the thermal gradient. Figure 3 compares friction factors calculated from Equation (18) with $(du^+/dy^+) = 0.41$ and for a Newtonian fluid friction factor corresponding to the Reynolds number from Equation (19). Calculated friction factors are observed to average about 4% less than the experimental data.

Power law non-Newtonian heat transfer with significant resistance in the transition region should be predicted by Equation (5). The average apparent viscosity for the transition region is assumed as the average between the wall viscosity and the viscosity at $y^+ = 12.1$. Thus,

$$\mu_T = \mu_{aw}\,[1 + (0.41)]^{n-1}/2 \qquad (20)$$

Figure 4 shows the Friend and Metzner heat transfer data with the flow behavior index range of 0.60 to 0.92. Combination of Equations (5) and (20) appear to give a reasonable representation of the experimental data. The data were corrected for the boundary layer resistance in accordance with Equation (15). The apparent viscosity at $y^+ = 12.1$ is assumed to represent the bulk fluid viscosity.

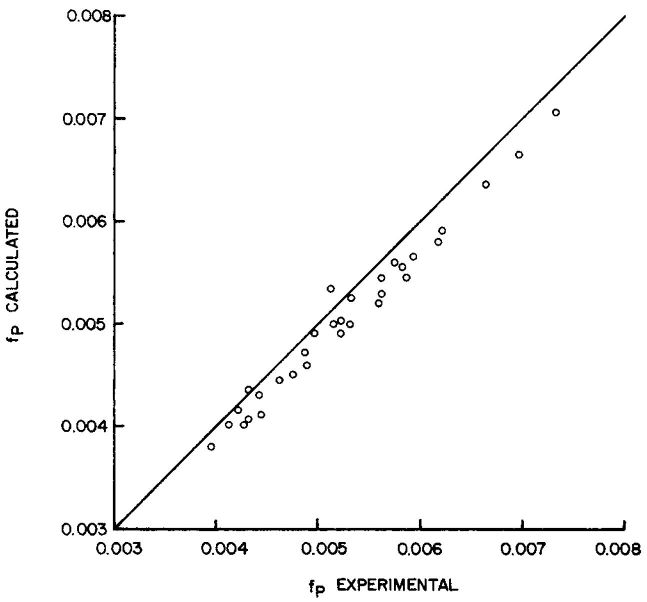

Fig. 3. Calculated and experimental non-Newtonian fluid friction factor.

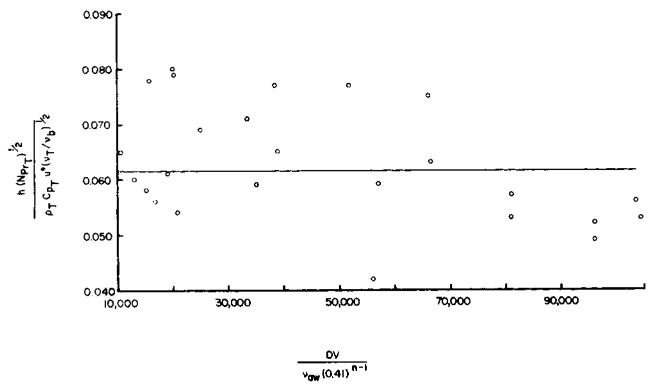

Fig. 4. Non-Newtonian heat transfer data.

NOTATION

C_p = specific heat
D = pipe diameter
f = friction factor
h = heat transfer coefficient
K = consistency index
k = mass transfer coefficient
k^+ = k/u^*
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{Sc} = Schmidt number
n = flow behavior index
t = eddy contact time
U_∞ = free stream velocity
U_∞^+ = U_∞/u^*
u = velocity in axial direction
u^* = shear velocity
u^+ = u/u^*
V = bulk fluid velocity
x = length in axial direction
y = radial distance from pipe wall
y^+ = yu^*/ν

Greek Letters

ϵ = eddy viscosity
μ = fluid viscosity
μ_{aw} = apparent viscosity evaluated at wall shear stress
ν = kinematic viscosity
ρ = fluid density
τ = shear stress at wall

Subscripts

a = apparent viscosity
b = bulk
BL = boundary layer
ET = eddy, transition region
EW = eddy, boundary layer region
i = isothermal
N = Newtonian fluid
ni = nonisothermal
P = power law non-Newtonian fluid
T = transition region
w = wall region
ws = wall surface

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